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PRESSURE STABILIZED BEAM FINITE ELEMENT

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Earl C. Steeves

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The development of a finite element for a pressure stabilized beam is presented and its adaptation to a computer code for the nonlinear structural analysis of frame-supported tents is described. The finite element is based on theory developed in earlier work and includes transverse shear and differential stiffness effects. Third order polynomials are used to represent the transverse displacements and bending rotations. First order polynomials are used to represent the axial and torsional deformation. A condensation is carried out to reduce the number —

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20. Abstract (cont'd)

of generalized coordinates to twelve. Two check cases are presented to establish the accuracy of the finite element.

PREFACE

This work is part of an ongoing investigation of the behavior of pressurized structural elements under load with the ultimate objective of making possible the use of such elements in the support structure of Army tents. The work was funded under the in-House Laboratory Independent Research program as a work unit entitled, "Study of the Stability of Pressure Stabilized Arches and their Structural Assemblies." In the reference citations the organizations "US Army Natick Laboratories" and "US Army Natick Development Center" refer to the organization now called the "US Army Natick Research and Development Command."

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PRESSURE STABILIZED BEAM FINITE ELEMENT

INTRODUCTION

A systems analysis of Army needs for shelters in the 1985 time frame revealed that the requirements for lightweight tentage of low packaged bulk with minimum setup and disassembly time might be most effectively achieved by a tent with a frame of highly pressurized (compared with present air-supported tents) structural elements supporting a lightweight fabric barrier, as illustrated in Figure 1. Since the structural elements are pressure stabilized, they can be fabricated from lightweight, flexible materials such as cloth, thus reducing the weight and bulk of the transport configuration. The use of bladders or coated fabric for these elements should provide sufficient air retention capability to eliminate the need for the dedicated air supply used with current air-supported tents.

To successfully develop this concept, a structural design capability was needed. To meet this need an investigation of the structural behavior of pressure-stabilized beams and arches was carried out and reported in references 1, 2, and 3. These references give the development of theories with experimental confirmation to predict the structural behavior of pressure stabilized beams and arches as a function of geometry, pressure level and material properties. The theories give adequate predictions of both the deformation and the load-carrying capability. These theories thus serve to characterize the behavior of the structural elements and can be used as a guide to design.

Since these structural elements are to be used in structural frames for tents, a more complete design capability is needed; that is, the capability to predict the deformation and load-carrying capability of frame assemblies made of pressure-stabilized elements and loaded through a fabric barrier. Thus, a structural analysis method capable of treating the interaction of assemblages of frame elements and the interaction of this frame with the fabric barrier is needed. The finite element procedure is well suited to such calculations, and a computer program using that procedure for the analysis of conventional frame tents was available. This program, which is described in references 4 and 5, includes a

1. Earl C. Steeves; A Linear Analysis of the Deformation of Pressure Stabilized Beams; US Army Natick Laboratories; Technical Report 75–47–AMEL, 1975 (AD A006493).

2. Earl C. Steeves; Behavior of Pressure Stabilized Beams Under Load; US Army Natick Development Center; Technical Report 75–82–AMEL; 1975 (AD A010702).

3. Earl C. Steeves; The Structural Behavior of Pressure Stabilized Arches; US Army Research & Development Command; Natick/TR–78/018; 1978.

4. Paul J. Remington, John C. O'Callahan and Richard Madden; Analysis of Stresses and Deflections in Frame Supported Tents; US Army Natick Laboratories; Technical Report 75–31; 1974 (AD A002072).

5. Paul J Remington, John C. O'Callahan and Richard Madden; Finite Element Analysis of Scale-Model Frame-Supported Tents; US Army Natick Research & Development Command; Technical Report 76–21–AMEL, 1975.

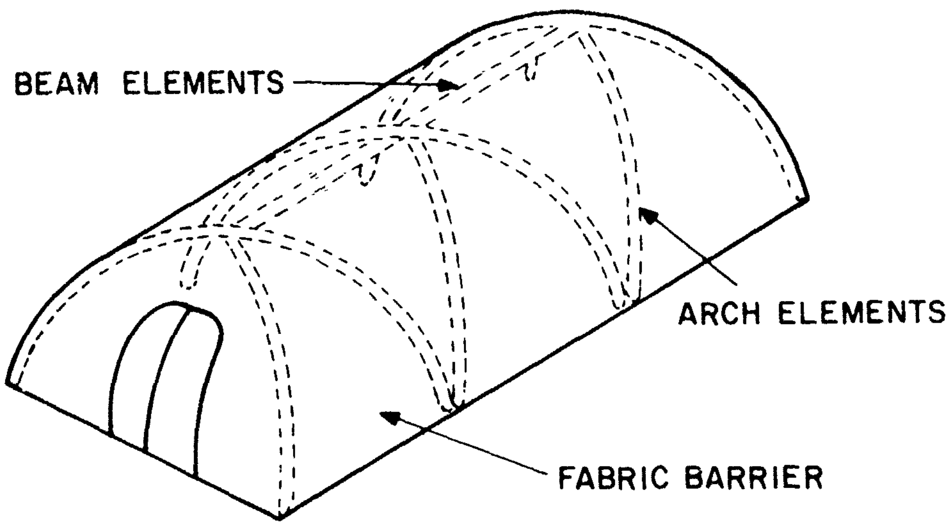

Figure 1. Tent Concept Using Pressure Stabilized Structural Elements

conventional beam element and a nonlinear membrane element for modeling the fabric barrier. Since a pressure stabilized beam element was not included, it was decided to develop such an element and adapt it to this existing computer program. The purpose here is to report this work. This will include the development of the finite element, its adaptation to the computer code and some check cases.

DEVELOPMENT OF FINITE ELEMENT

We proceed here with the derivation of the finite element with the intended result a 12 X 12 stiffness matrix associated with generalized coordinate set comprised of the three deflections and three rotations of each of the beam end points. The basis for this derivation is the energy principle derived in reference 1. This energy principle which includes bending in one plane and axial deformation is augmented to include bending in two planes, axial deformation, and torsion. The displacement fields are represented by polynomials in the coordinate specifying position along the beam.

Because of the number and order of polynomials used, the number of generalized coordinates exceeds 12, so a condensation is carried out.

In carrying out this derivation we use the coordinate system and sign convention given in Figure 2. Shown are the sign conventions associated with the basic theory and that associated with the finite element. The potential energy expression to be used is that given in reference 1 and augmented as indicated above and is expressed in dimensional form as:

$$\int_0^L \tfrac{1}{2} \Big\{ E\Big(\frac{du}{dx}\Big)^2 + D\Big(\frac{d\phi}{dx}\Big)^2 + D\Big(\frac{d\psi}{dx}\Big)^2 + G\Big(\frac{d\theta}{dx}\Big)^2 + C\Big(\frac{dv}{dx} - \psi\Big)^2 + C\Big(\frac{dW}{dx} + \phi\Big)^2 + P\Big(\frac{dW}{dx}\Big)^2 + P\Big(\frac{dv}{dx}\Big)^2 \Big\} dx \qquad (1)$$

This expression is based on the assumptions of uniaxial strain and linear expression for the shear strain in terms of the displacements. Since we seek a finite element with which forces will be applied at the nodes, the distributed forces are not included in (1). The potential energy expression is a quadratic in the displacements and can be written in matrix form as

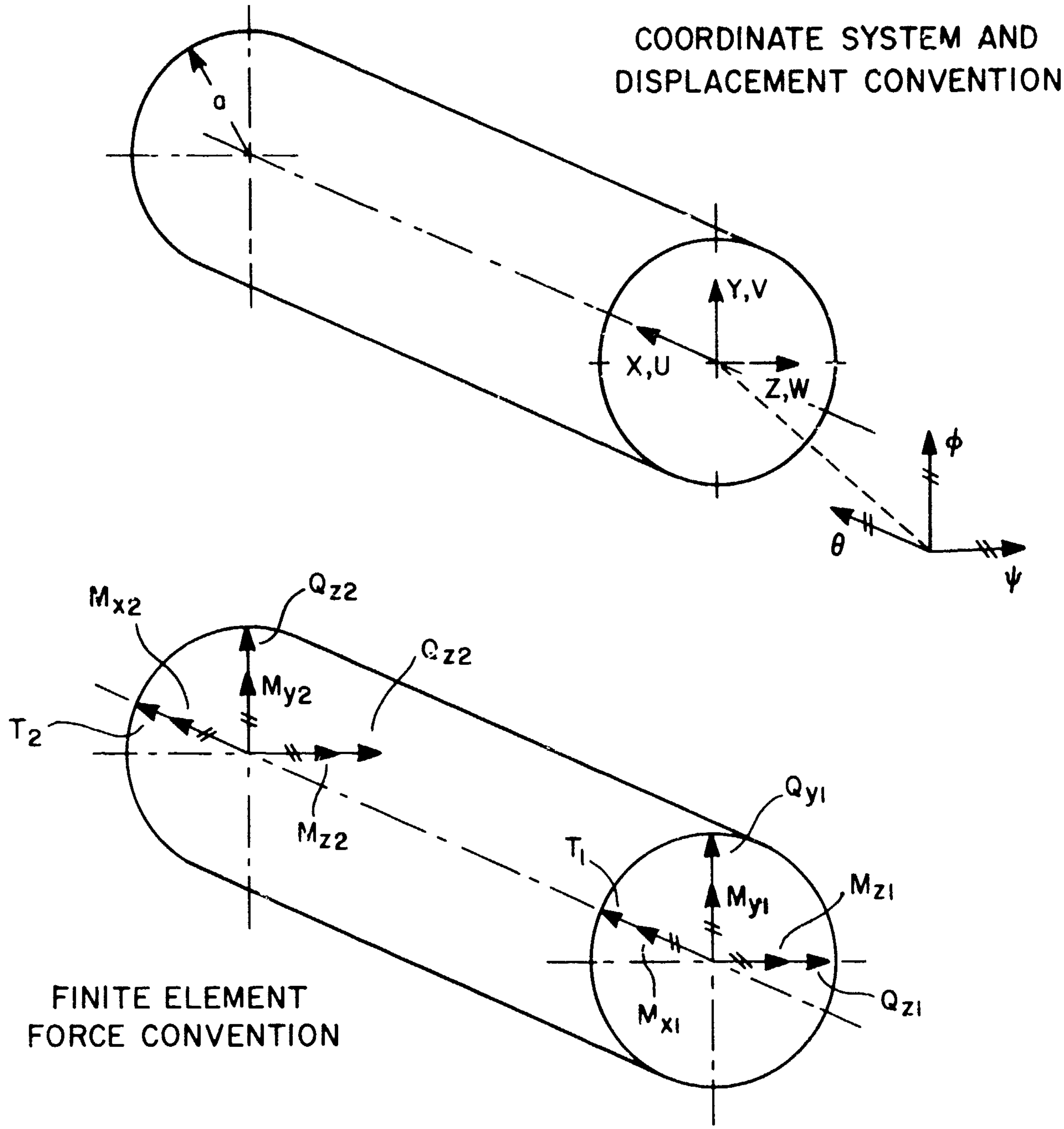

FIGURE 2. COORDINATE SYSTEM AND SIGN CONVENTION

$$
\frac{1}{2}\int_0^L \begin{bmatrix} \frac{du}{dx} & \frac{d\phi}{dx} & \frac{dW}{dx} & \phi & \frac{d\theta}{dx} & \frac{d\Psi}{dx} & \frac{dv}{dx} & \Psi \end{bmatrix}
\begin{bmatrix}
E & & & & & & & \\
& D & & & & & & \\
& & C+P & C & & & & \\
& & C & C & & & & \\
& & & & G & & & \\
& & & & & D & & \\
& & & & & & C+P & -C \\
& & & & & & -C & C
\end{bmatrix}
\begin{Bmatrix} \frac{du}{dx} \\ \frac{d\phi}{dx} \\ \frac{dW}{dx} \\ \varphi \\ \frac{d\theta}{dx} \\ \frac{d\Psi}{dx} \\ \frac{dv}{dx} \\ \Psi \end{Bmatrix} dx \quad (2)
$$

where

$$E = 2\pi a C_{11} \qquad D = \pi a^3 C_{11}$$

$$C = \pi a C_{33} \qquad G = \pi a^3 C_{33}$$

$$P = \pi a^2 p/2$$

In these definitions C_{11}, C_{33}, and p are respectively the elastic modulus, the shear modulus, and the pressure. It can easily be seen from this espression that the axial, torsional, and two bending energies are uncoupled and thus can be treated independently informulating the finite element stiffness matrix.

The energy resulting from axial deformation is given by the expression $E(\frac{du}{dx})^2$, which is the classical expression. With the use of a linear representation for the axial displacement field in terms of the end point axial displacements, U_1 and U_2, the following energy expression is obtained upon integration:

$$
\frac{1}{2} \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} E/L & -E/L \\ -E/L & E/L \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} \quad (3)
$$

This is the classical stiffness matrix for linear axial behavior.

Similarly, the torsional energy is given by $G(\frac{d\theta}{dx})^2$ and a representation of θ linear in the x coordinate in terms of the end point rotation θ_1 and θ_2, yields for the energy

$$\tfrac{1}{2}\begin{bmatrix}\theta_1 & \theta_2\end{bmatrix}\begin{bmatrix} G/L & G/L \\ -G/L & G/L \end{bmatrix}\begin{Bmatrix}\theta_1 \\ \theta_2\end{Bmatrix} \tag{4}$$

Examination of (2) reveals that the two bending problems differ only in the algebraic signs of the off diagonal elements. This being so, we can derive the stiffness matrix associated with one of the bending problems and deduce the other form it. We will treat the problem described by W and φ. The formulation of the bending stiffness matrix is not as direct or simple as the above problems or the classical bending problem. This results from the use of two independent parameters, W and φ, to describe the bending problem. The use of four generalized coordinates is generally sufficient to approximately describe the beam bending problem and is though to be adequate here also. However, dividing these four coordinates between the two independent variables, W and φ, only allows for the use of linear approximation of these variables within the context of polynomial approximation. That is not believed to be an adequate approximation, since for W it only allows description of rigid body displacements. It was, therefore, decided to use two third order polynomials, one to represent W and the other to represent φ. This gives a stiffness matrix in terms of eight generalized coordinates, so a condensation is performed to reduce the number to four. The bending energy is

$$\tfrac{1}{2}\int_0^L \begin{bmatrix}\dfrac{d\varphi}{dx} & \dfrac{dW}{dx} & \varphi\end{bmatrix}\begin{bmatrix} D & O & O \\ O & C+P & C \\ O & C & C\end{bmatrix}\begin{Bmatrix}\dfrac{d\varphi}{dx} \\ \dfrac{dW}{dx} \\ \varphi\end{Bmatrix} dx \tag{5}$$

and the polynomial approximations are

$$\begin{Bmatrix}\dfrac{d\varphi}{dx} \\ \dfrac{dW}{dx} \\ \varphi\end{Bmatrix} = \begin{bmatrix} O & h_1' & O & -h_1' & O & h_3' & O & h_4' \\ h_1' & O & -h_1' & O & h_3' & O & h_4' & O \\ O & h_1 & O & h_2 & O & h_3 & O & h_4 \end{bmatrix}\begin{Bmatrix} W_1 \\ \varphi_1 \\ W_2 \\ \varphi_2 \\ W_1' \\ \varphi_1' \\ W_2' \\ \varphi_2'\end{Bmatrix} \tag{6}$$

In equation (6)

$$
\begin{aligned}
h_1 &= 1 - 3x^2/L^2 + 2x^3/L^3 & \qquad h_1' &= 6x^2/L^3 - 6x/L^2 \\
h_2 &= 3x^2/L^2 - 2x^3/L^3 & \qquad h_3' &= 1 - 4x/L + 3x^2/L^2 \\
h_3 &= x - 2x^2/L + x^3/L^2 & \qquad h_4' &= 3x^2/L^2 - 2x/L \\
h_4 &= x^3/L^2 - x^2/L & &
\end{aligned}
\tag{7}
$$

The elements of the vector of order eight in equation (6) are the transverse deflection and cross-section rotation and their derivations evaluated at $x = 0$ and $x = L$. The subscript 1 denotes evaluation at $x = 0$, and subscript 2 denotes evaluation at $x = L$. The prime denotes the derivative of the variable. Substitution of (6) into (5) and integration yields the following energy expression:

$$
\tfrac{1}{2} \begin{bmatrix} \Delta_1^t & \Delta_2^t \end{bmatrix} \begin{bmatrix} K_{11} & K_{12} \\ K_{12}^t & K_{22} \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} \tag{8}
$$

where

$$
\Delta_1 = \begin{Bmatrix} W_1 \\ \phi_1 \\ W_2 \\ \phi_2 \end{Bmatrix} \qquad \Delta_2 = \begin{Bmatrix} W_1' \\ \phi_1' \\ W_2' \\ \phi_2' \end{Bmatrix} \tag{9}
$$

The elements of the matrices K_{11}, K_{12} and K_{22} are given in Appendix A. We have an eighth-order stiffness matrix which we would like to reduce to fourth order to make it compatible with other finite elements for beam bending but particularly for compatibility with the finite element described in references 4 and 5. To accomplish this the set of generalized coordinates contained in Δ_1 are chosen to be the set to be used and those in Δ_2 the set to be removed. It is also assumed that the applied forces do work only through the generalized coordinates Δ_1 so that the potential energy including the external forces can be written

$$
\tfrac{1}{2} \begin{bmatrix} \Delta_1^t & \Delta_2^t \end{bmatrix} \begin{bmatrix} K_{11} & K_{12} \\ K_{12}^t & K_{22} \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} - \begin{bmatrix} \Delta_1^t & \Delta_2^t \end{bmatrix} \begin{Bmatrix} Q \\ 0 \end{Bmatrix} \tag{10}
$$

In this situation the following transformation of coordinates can be used to carry out the desired reduction or condensation

$$\begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} = \begin{bmatrix} I \\ -K_{22}^{-1} \; K_{12}^{t} \end{bmatrix} \begin{Bmatrix} \Delta_1 \end{Bmatrix} \tag{11}$$

with I denoting the identity matrix. Substitution of (11) into (10) gives for the energy expression

$$\tfrac{1}{2} \quad \Delta_1^t S \Delta_1 \quad - \quad \Delta_1^t Q \tag{12}$$

Where S is the desired 4 x 4 stiffness matrix expressed in terms of the original submatrices as

$$S = K_{11} - K_{12} \; K_{22}^{-1} \; K_{12}^{t} \tag{13}$$

This completes the development of the stiffness matrices for axial, torsional, and bending deformation and the complete 12 x 12 stiffness matrix associated with the end point displacements and rotations can be assembled. In doing so, attention is paid to the order of the generalized coordinates or end point displacements used in the NONFESA (Nonlinear Finite Element Structural Analysis) program in which this finite element will be used. The desired stiffness matrix appears as follows in the expression for the potential energy:

$$\tfrac{1}{2} \begin{bmatrix} U_1 & V_1 & W_1 & \theta_1 & \varphi_1 & \psi_1 & U_2 & V_2 & W_2 & \theta_2 & \varphi_2 & \psi_2 \end{bmatrix} \begin{bmatrix}
E/L & & & & & & -E/L & & & & & \\
 & S_{11} & & & & S_{12} & & S_{13} & & & & S_{14} \\
 & & S_{11} & & -S_{12} & & & & S_{13} & & -S_{14} & \\
 & & & G/L & & & & & & -G/L & & \\
 & & -S_{12} & & S_{22} & & & & -S_{23} & & S_{24} & \\
 & S_{12} & & & & S_{22} & & S_{23} & & & & S_{24} \\
-E/L & & & & & & E/L & & & & & \\
 & S_{13} & & & & S_{23} & & S_{33} & & & & S_{34} \\
 & & S_{13} & & -S_{23} & & & & S_{33} & & -S_{34} & \\
 & & & -G/L & & & & & & G/L & & \\
 & & -S_{14} & & S_{24} & & & & -S_{34} & & S_{44} & \\
 & S_{14} & & & & S_{24} & & S_{34} & & & & S_{44}
\end{bmatrix} \begin{Bmatrix} U_1 \\ V_1 \\ W_1 \\ \theta_1 \\ \varphi_1 \\ \psi_1 \\ U_2 \\ V_2 \\ W_2 \\ \theta_2 \\ \varphi_2 \\ \psi_2 \end{Bmatrix} \tag{14}$$

The elements s_{ij} in (14) are the elements of the matrix S given in (13). In assembling this matrix, account has been taken of the algebraic sign difference of the off diagonal elements in the bending problem associated with V and ψ that was discussed previously in connection with expression (2). The matrix in (14) is the desired stiffness matrix, and having it we can proceed with its adaptation to the NONFESA.

In carrying out this adaptation, two options were open, the first being to create an entirely new element to be included along with the beam, membrane and truss elements already in the program, and the second being to modify the present beam element by making the pressure stabilized beam element an option on the conventional beam element. Because of the great similarity of this beam element to the conventional one, this second option was chosen. To accomplish this adaptation changes were required in both the program and the input. Program changes were required in the computation of the stiffness matrix and in the computation of the stresses for output. The beam stiffness matrix is computed in the subroutine named NEWBM, and in this subroutine an option was added for calculation of the stiffness matrix according to (14). This option is controlled by the parameter JPRES. For the calculation of the stiffness matrix according to (14) two additional subroutines are needed, ELB to calculate the stiffness matrix S and GJSOL to solve a system of equations by Gauss-Jordan reduction. In ELB the order eight stiffness matrix of (8) is calculated according to the formulas for the elements given in the appendix. The condensation of the matrix is carried out using subroutine GJSOL, and the order four matrix is returned to subroutine NEWBM for assembly into the 12 x 12 stiffness matrix along with the other elements which are calculated directly into the matrix. The reader may question the need for inclusion of the subroutine GJSOL for Gauss–Jordan reduction when one is already present in NONFESA for solution of the large system of equations generated by the program. The subroutine already included is especially designed for large systems of equations stored on tape in block format so its use on a fourth order system seemed inefficient and overly complex. Thus a very simple 25-line subroutine was added to carry out the condensation. The other programming changes are in subroutine BEAM where the stresses are computed for output from the nodal forces. Because the pressure-stabilized beam is a circular cylindrical membrane, membrane-type stresses are computed as follows:

> The uniform stress resulting from axial deformation is computed by dividing the axial nodal forces by the circumference of the cross-section.
>
> The maximum and minimum stresses resulting from bending deformation are computed for each of the two planes of bending. The magnitudes of these parameters are the quantities and their negatives obtained by dividing the nodal moment for each of the planes of bending by the area inclosed by the cross-section.
>
> The average stresses from shear deformation are computed by dividing nodal shear force associated with each of the two planes of bending by the circumference of the cross-section.

These stress calculations are carried out for both ends of the beam.

The only input that changes is that associated with the beam element, with these changes being on the material property card, the element property card and the beam data cards of reference 6. For the pressure stabilized beam the needed input in addition to the control parameter is the beam length, the elastic modulus, Poisson's Ratio, the cross-section radius, and the inflation pressure. The beam length is computed from the beam end point nodal coordinates and this does not constitute a change. The elastic modulus and Poisson's Ratio are entered with the material property card, with the elastic modulus being the first entry following the identification number and Poisson's Ratio the second. The elastic modulus required is of the membrane type having the units of force per unit length. If Poisson's Ration is known, it can be entered, but typically the parameter that is known is the membrane shear modulus. In the program the shear modulus is computed from the elastic modulus and Poisson's Ratio according to the relation

$$C_{33} = C_{11}/2(1 + \mu) \tag{15}$$

Thus, when the shear modulus and the elastic modulus are known, the Poisson's Ratio to be entered as the second entry on the material property card should be calculated by the expression by solving (15) for μ in terms of C_{11} and C_{33}. The element property card contains an identification number followed by eight numerical parameters. For the pressure-stabilized beam elements, the first two of the numerical parameters are the cross-section radius and the inflation pressure, in that order. The fifth of these parameters should be entered as a negative number as this is used to control the option on the computation of the stresses from the nodal forces. The remaining five parameters are dummies for the pressure-stabilized beam element, and nonvanishing entries should be made for them. Control of the pressure-stabilized beam option is accomplished by the parameter JPRES on the beam data card (there is one of these cards for every beam) If a given beam is to use the pressure-stabilized beam option the PJRES must be entered as unity in column 68 of the beam data card, otherwise it is to be left blank or entered as zero. If the pressure-stabilized beam element is exercised then the material property and element property numbers entered on the beam data card must correspond to the identification numbers corresponding to material and element property sets for pressure-stabilized beams as described above.

6 John C. O'Callahan NONFESA – Nonlinear Finite Element Structural Analysis Code for the Analysis of Stresses and Deflections in Frame Supported Tents, Bolt Beranek and Newman, Inc., Report No. 2803, 1975.

CHECK CASES

To establish the validity of the pressure stabilized beam finite element two check cases are presented here. A straight beam loaded at its midpoint in two perpendicular planes and a circular arch loaded in its plane are the check cases. The finite element calculations are compared with the exact solutions of the theory given in references 4 and 5.

The simplest structure that can be modeled using the pressure stabilized beam finite element is a straight beam, as shown in Figure 3. We have taken a beam 120 cm long having a circular cross section of 3 cm radius and divided it into six elements of equal length. The element numbers in Figure 3 are inclosed in the square symbols and the nodes are numbered one through 8 and denoted by the solid circular symbols. Node eight is a dummy node require to establish the element coordinate system. The global coordinate system is shown in Figure 3, and for this case local coordinate system for each of the elements coincides with the global system. The ends of the beam are simply supported and restrained against rotation about the global x-axis. Concentrated loads are applied at the center of the beam, node number 4, in the Y and Z directions. The beam material properties and the inflation pressure are also tabulated in Figure 3.

It is desirable to be able to model curved members with this finite element, so to establish its validity for this class of structures, a circular arch was modeled as shown in Figure 4. This arch has the form of a semicircle with a radius of 120 cm and the circular cross section has a radius of 4 cm. This arch is modeled with eighteen elements of equal length. The element members are inclused in square symbols and the node points are denoted by the solid circular symbols. Node number twenty is a dummy used to establish the local element coordinate system. The global coordinate system is shown in Figure 4 and the arch material properties and inflation pressure are tabulated there also. The ends of the arch are simply supported, and displacements in the X direction for all nodes are suppressed. A concentrated load of unit magnitude is applied at node number ten in the negative Z direction.

For both the beam and the arch we present the input data and the output results from the NONFESA program and a comparison of the finite element results with the results from the exact solutions of the theory. The computer input and results associated with the NONFESA program are given in Appendices B and C for the beam and arch respectively and the comparison of the finite element results with those from the exact solution are given in Table 1 and 2. The input data in Appendices B and C are presented as illustrative examples and no comments are needed. The output from the NONFESA program also appearing in Appendices B and C can be divided into two parts, the first being a printout of the input data and some computed setup data such as the correspondence between degrees of freedom and equation numbers. It should be noted that the cross-section radius and the inflation pressure are printed under the Beam Geometric Properties heading as the Area X and Area Y, respectively. Also note that the parameter Inertia Y under this same heading is a negative number. As discussed above

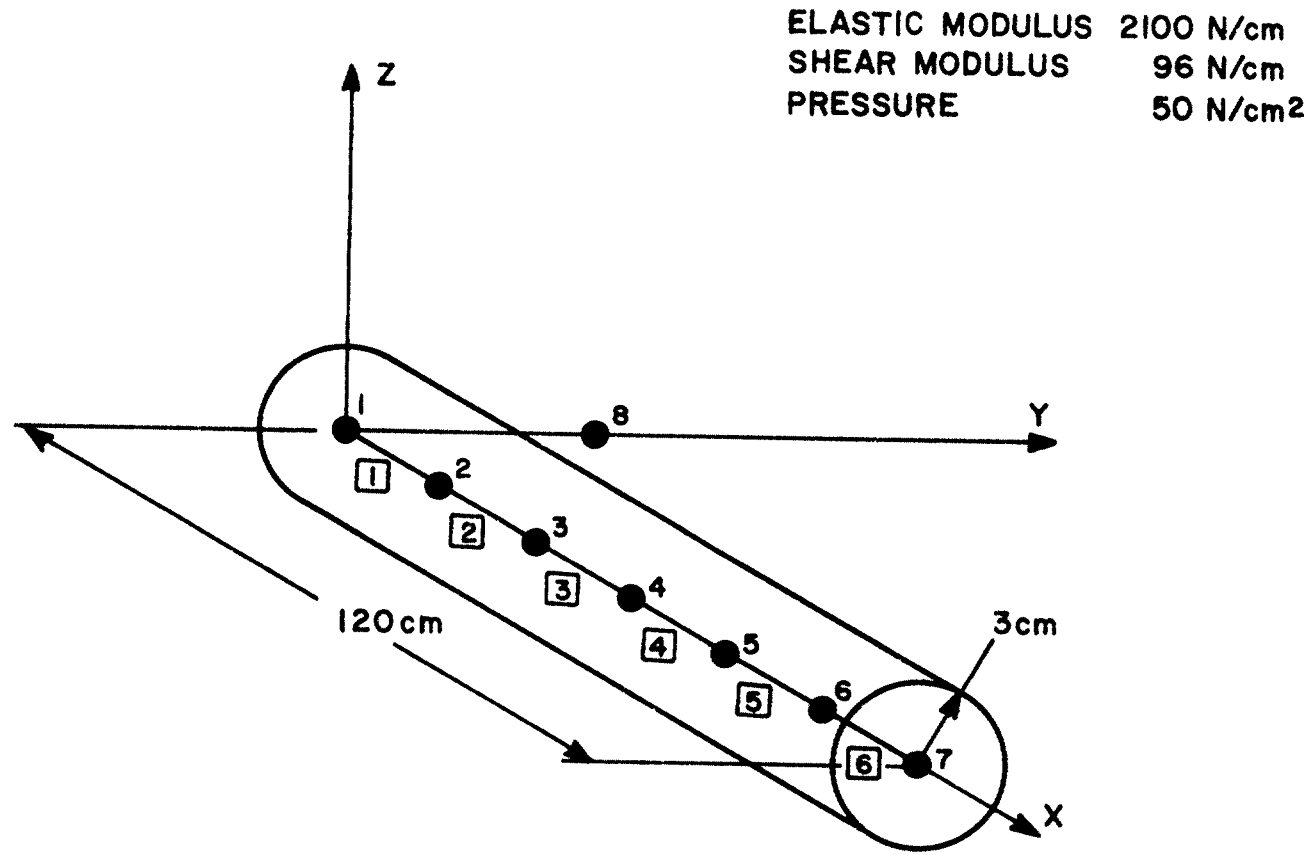

FIGURE 3. FINITE ELEMENT MODEL FOR STRAIGHT BEAM CHECK CASE

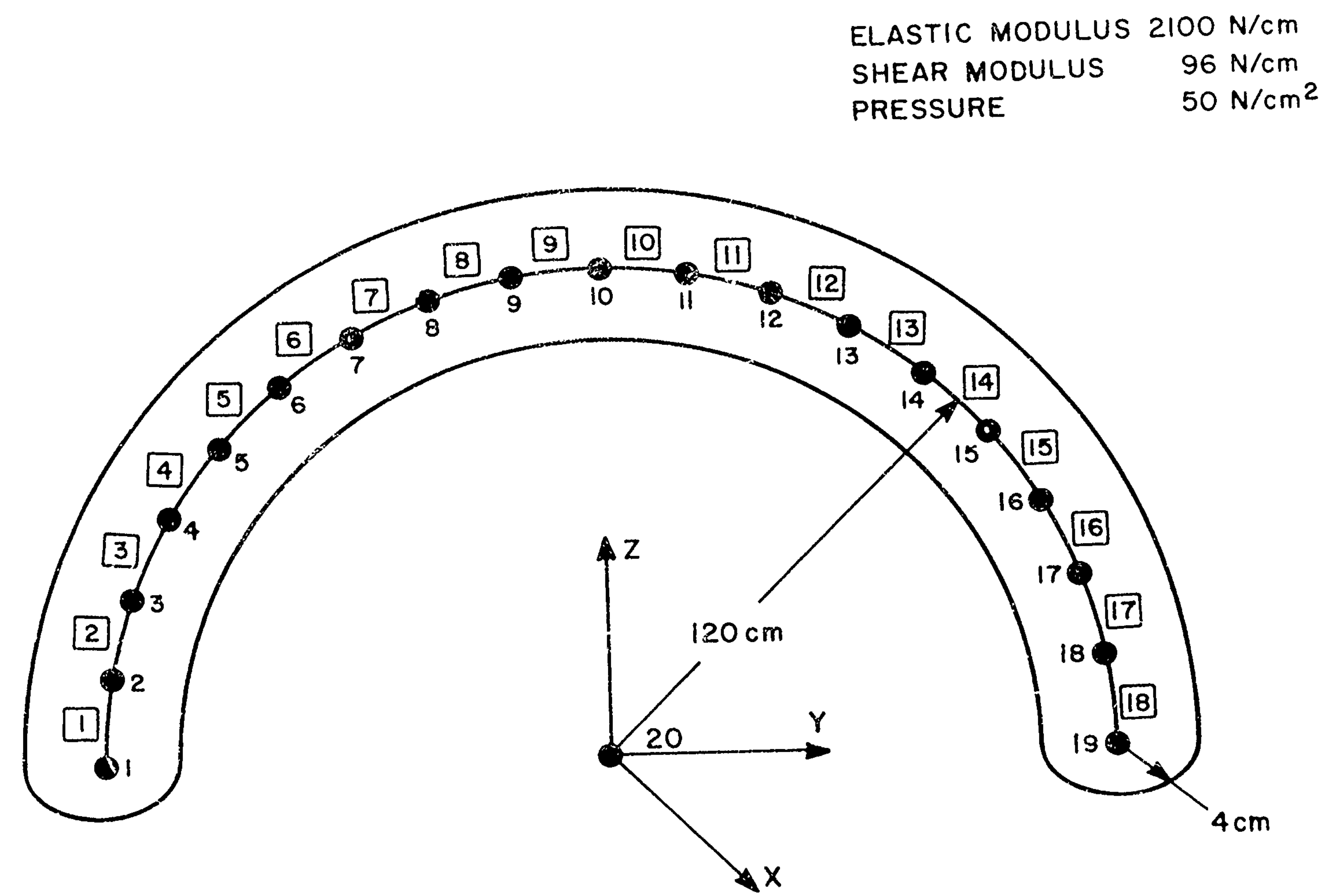

FIGURE 4. FINITE ELEMENT MODEL FOR CIRCULAR ARCH CHECK CASE

TABLE 1

Comparison of Finite Element Results with Those from the Exact Solution for the Beam Check Case

Position (cm)	Node Number	Displacement: Finite Element	Displacement: Exact	Rotation: Finite Element	Rotation: Exact	Bending Stress: Finite Element	Bending Stress: Exact
−60	1	0.	0.	-3.2×10^{-4}	-3.2×10^{-4}	0.	0
−40	2	0.0071	0.0071	−2.9	−2.9	0.0215	0.0215
−20	3	0.0137	0.0137	−2.1	−2.1	0.0561	0.0561
0	4	0.0189	0.0189	0.	0.	0.125	0.125
20	5	0.0137	0.0137	2.1	2.1	0.0561	0.0561
40	6	0.0071	0.0071	2.9	2.9	0.0251	0.0251
60	7	0.	0.	3.2	3.2	0	0

TABLE 2

Comparison of Finite Element Results with Those from the Exact Solution for the Arch Check Case

Angular Position Degrees	Node No.	Normal Displacement cm: Finite Element	Normal Displacement cm: Exact	Axial Displacement cm: Finite Element	Axial Displacement cm: Exact	Rotation: Finite Element	Rotation: Exact
–90	1	0.	0.	0.	0.	1.4X10^{-4}	1 4X10^{-4}
–80	2	– 3.58X10^{-3}	– 3.58X10^{-3}	–0.52X10^{-3}	–0.53X10^{-3}	1.3	1 2
–70	3	– 5.82	– 5.81	–1.57	–1.59	0.7	0.7
–60	4	– 6.39	– 6.35	–2.86	–2.91	0 06	0.05
–50	5	– 5.16	– 5.11	–4.11	–4.16	–0.6	–0.6
–40	6	– 2.19	– 2.12	–4.98	–5.03	–1.3	–1.3
–30	7	2.28	2.39	–5.19	–5.24	–1.8	–1.8
–20	8	7.89	8.0	–4.49	–4.54	–2.0	–2.0
–10	9	13.5	14.0	–2.17	–2.78	–1.6	–1.6
0	10	19.2	19.3	0.	0.	0.	0.

Angular Position Degrees	Node No.	Axial Force, N: Finite Element	Axial Force, N: Exact	Moment N-cm: Finite Element	Moment N-cm: Exact	Stress N/cm: Finite Element	Stress N/cm: Exact
–90	1	– 0.525	– 0.500	0.0	0 0	–0.020	–0.019
–80	2	– 0.543	– 0.545	0.895	0.868	–0 039	–0.039
–70	3	– 0.572	– 0.573	1.31	1.27	–0.049	–0.049
–60	4	– 0.583	– 0.585	1.47	1.43	–0.052	–0.052
–50	5	– 0.576	– 0.578	1.46	1.41	–0.052	–0 051
–40	6	– 0.552	– 0.554	1.29	1.24	–0.047	–0.047
–30	7	– 0.511	– 0.513	0.88	0.836	–0.037	–0.037
–20	8	– 0.455	– 0.456	0.046	0.007	–0.019	–0.018
–10	9	– 0.385	– 0.386	–1.66	–1.69	–0.018	0.019
0	10	– 0.347	– 0.304	–5.26	–5.29	–0.091	0.095

under the description of input, this is used for testing whether the beam element is a pressure stabilized element for computation of stresses. The second part of the output contains the computed results for the beam stresses, beam forces and moments, nodal displacements and rotations, and the nodal displacements. As is indicated on the printout, for pressure stabilized beam elements the stress should be interpreted as a membrane stress with units of force per unit length.

The comparison of the finite element results with those given by the exact solution of the beam theory of reference 1 is presented in Table 1 The simple six-element finite element model is in agreement with the exact solution to all decimal places shown for all three parameters, displacement, rotation, and bending stress. The limit on the number of places given in Table 1 is the output of the NONFESA program. The maximum possible percentage difference between the finite element results and the exact solution is then 2 4%. This is good agreement and establishes the validity of the element for straight beams

The comparison of the finite element results with those given by the exact solution of the arch theory of reference 3 is presented in Table 2. Results are presented for half the arch since the results for the other half can be inferred from those presented by the symmetry of the structure and the loading. The finite element results generally follow the results from the exact solution with differences of less than 5% and in most cases considerably less than 5%. There are, however, three cases where the difference is greater than 5% Two of these, the axial displacement at node 9 and the axial force at node 10, do not appear to have an easy explanation but the third one, the moment at node 8, appears to be due to the closeness of the node to the point where the moment changes sign In this case, the exact solution is very close to the value zero, and thus any error in the finite element solution is magnified. In general, this comparison is good and establishes the validity of the finite element for curved structures.

CONCLUDING REMARKS

The development of a finite element for pressure-stabilized beams is described along with its adaptation to the Nonlinear Finite Element Structural Analysis program. This provides a means for carrying out design analysis of pressure-stabilized frame-supported tents under rather general loading. The validity of the finite element and the correctness of its adaptation to the structural analysis program is established by the comparison of finite element results with exact solution for a straight beam and a circular arch.

REFERENCES

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4. Remington, Paul J., John C. O'Callahan and Richard Madden; Analysis of Stresses and Deflections in Frame Supported Tents; US Army Natick Laboratories, Technical Report 75–31; 1974 (AD A002072).

5. Remington, Paul J., John C. O'Callahan and Richard Madden; Finite-Element Analysis of Scale-Model Frame-Supported Tents; US Army Natick Research & Development Command, Technical Report 76–21–AMEL; 1975.

6. O'Callahan, John C.; NONFESA -- Nonlinear Finite Element Structural Analysis Code for the Analysis of Stresses and Deflections in Frame Supported Tents; Bolt Beranek and Newman, Inc., Report No. 2803, 1975.

APPENDIX A

ELEMENTS OF THE 8 X 8 BENDING STIFFNESS MATRIX

APPENDIX A

ELEMENTS OF THE 8 x 8 BENDING STIFFNESS MATRIX

The elements of the 8 x 8 bending stiffness matrix are given here in terms of the 4 x 4 submatrices K_{11}, K_{12}, and K_{22}. In presenting these elements, use is made of the symmetry of the matrices wherever possible

Submatrix K_{11}

Row index	Column index	Element
1	1	6(P + C)/5L
2	1	C/2
3	1	−6(P + C)/5L
4	1	C/2
2	2	6D/5L + 13CL/35
3	2	−C/2
4	2	−6D/5L + 9CL/70
3	3	6(P + C)/5L
4	3	−C/2
4	4	6D/5L + 13CL/35

Submatrix K_{12}

Row index	Column index	Element
1	1	(P + C)/10
2	1	−CL/10
3	1	−(P + C)/10
4	1	CL/10
1	2	CL/10

Row index	Column index	Element
2	2	$D/10 + 11CL^2/210$
3	2	$-CL/10$
4	2	$-D/10 + 13CL^2/420$
1	3	$(P + C)/10$
2	3	$CL/10$
3	3	$-(P + C)/10$
4	3	$-CL/10$
1	4	$-CL/10$
2	4	$D/10 - 13CL^2/420$
3	4	$CL/10$
4	4	$-D/10 - 11CL^2/210$

Submatrix K_{22}

Row index	Column index	Element
1	1	$2(P + C)L/15$
2	1	0
3	1	$-(P + C)L/30$
4	1	$-CL^2/60$
2	2	$2DL/15 + CL^3/105$
3	2	$CL^2/60$
4	2	$-DL/30 - CL^3/140$
3	3	$2(P + C)L/15$
3	4	0
4	4	$2DL/15 + CL^3/105$

APPENDIX B

INPUT DATA AND RESULTS FOR BEAM CHECK CASE

INPUT DATA

```
PRESSURIZED BEAM CHECK CASE
 8    1
 1    1    1    1    1    0    00.0      0.0      0.0     1101000
 2    0    0    0    0    0    020.0     0.0      0.0
 3    0    0    0    0    0    040.0     0.0      0.0
 4    0    0    0    0    0    060.0     0.0      0.0
 5    0    0    0    0    0    080.0     0.0      0.
 6    0    0    0    0    0    0100.0    0.0      0.0
 7    1    1    1    1    0    0120.0    0.0      0.0
 8    1    1    1    1    1    10.0      10.0     0.0
 1    6    1    0    1
 1 2100.0   9.9375
 1 4.0      50.0      1.0        1.0       -1.0     1.0      1.0  1.0
 1    1    2    8    1    1    0    0
 2    2    3    8    1    1    0    0                                 1
 3    3    4    8    1    1    0    0                                 1
 4    4    5    8    1    1    0    0                                 1
 5    5    6    8    1    1    0    0                                 1
 6    6    7    8    1    1    0    0                                 1
 4     0.0          1.0       1.0       0.0      0.0      0.0        1
```

RESULTS

```
*************************************************
*                                               *
*         P R O G R A M   N O N F E S A         *
*                                               *
*************************************************
*                                               *
* NONLINEAR FINITE ELEMENT STRUCTURAL ANALYSIS  *
* PROGRAM FOR DETERMINING DEFLECTIONS AND       *
* STRESSES OF TENT STRUCTURES.  THE PROGRAM IS A *
* GENERAL PROGRAM CAPABLE OF ASSEMBLING MANY    *
* DIFFERENT ELEMENT TYPES.  THE PRESENT VERSION *
* (MARCH,1975) CONTAINS ELEMENTS TYPES-         *
*               1. LINEAR BEAM ELEMENT          *
*                  1.1 LINEAR BEAM              *
*                  1.2 MODIFIED BEAM            *
*               2. GEOMETRIC NONLINEAR MEMBRANE *
*                  2.1 LINEAR MATERIAL          *
*                  2.2 NONLINEAR MATERIAL       *
*                  2.3 MATERIAL REFERENCE PLANE *
*               3. LINEAR TRUSS ELEMENT         *
*                  3.1 TRUSS(PRE-LOAD)          *
*                  3.2 GUY LINE(TENSILE)        *
*                  3.3 COMPRESSIVE BAR          *
*                                               *
*************************************************
*                                               *
* PROGRAM DEVELOPED AT BOLT BERANEK AND NEWMAN  *
* CAMBRIDGE, MASS. BY J.C. OCALLAHAN.           *
*                                               *
*************************************************
```

```
PRESSURIZED BEAM CHECK CASE

NUMBER OF NODAL POINTS     =    8

NUMBER OF ELEMENT TYPES    =    1

MAXIMUM NUMBER OF CYCLES   =    0

NUMBER OF LOAD STEPS       =    0

CONVERGENCE TOLERANCE      =  .000

STARTING MEMBRANE TENSION=    .000

DERIVATIVE SPACING         =  .100+00

PRINT CONTROL
   (0--OFF, 1--ON, EXCEPT FOR DISPLACEMENTS)
   1. DELTA DISPLACEMENTS     = 1
   2. ELEMENT STRESSES        = 1
   3. BEAM FORCES             = 1
   4. DISPLACEMENTS(1--OFF)= 0
   5. NODAL POSITIONS         = 1
   6. DEBUG-MEMBRANE          = 0
        ELEMENT STIFFNESS
   7. DEBUG-MEMBRANE          = 0
        GEOMETRIC PROP
   8. DEBUG-MEMBRANE          = 0
        MATERIAL COEFS
```

NODAL POINT INPUT DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	NODAL POINT COORDINATES X	Y	Z		
1	1	1	1	1	0	0	.000	.000	.000	0	0
2	0	0	0	0	0	0	20.000	.000	.000	0	0
3	0	0	0	0	0	0	40.000	.000	.000	0	0
4	0	0	0	0	0	0	60.000	.000	.000	0	0
5	0	0	0	0	0	0	80.000	.000	.000	0	0
6	0	0	0	0	0	0	100.000	.000	.000	0	0
7	1	1	1	1	0	0	120.000	.000	.000	0	0
8	1	1	1	1	1	1	.000	10.000	.000	0	0

GENERATED NODAL DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	NODAL POINT COORDINATES X	Y	Z
	1	1	1	1	0	0	.000	.000	.000
2	0	0	0	0	0	0	20.000	.000	.000
3	0	0	0	0	0	0	40.000	.000	.000
4	0	0	0	0	0	0	60.000	.000	.000
5	0	0	0	0	0	0	80.000	.000	.000
6	0	0	0	0	0	0	100.000	.000	.000
7	1	1	1	1	0	0	120.000	.000	.000
8	1	1	1	1	1	1	.000	10.000	.000

EQUATION NUMBERS

N	X	Y	Z	XX	YY	ZZ
1	0	0	0	0	1	2
2	3	4	5	6	7	8
3	9	10	11	12	13	14
4	15	16	17	18	19	20
5	21	22	23	24	25	26
6	27	28	29	30	31	32
7	0	0	0	0	33	34
8	0	0	0	0	0	0

```
T H R E E   D I M E N S I O N A L   B E A M   E L E M E N T S

     NUMBER OF BEAMS                 =     6
     NUMBER OF GEOMETRIC PROPERTY SETS=    1
     NUMBER OF FIXED END FORCE SETS   =    0
     NUMBER OF MATERIALS              =    1
```

MATERIAL	YOUNG S MODULUS	POISSON S RATIO
1	.21000+04	9.93750

BEAM GEOMETRIC PROPERTIES

ELEMENT TYPE	AREA X	AREA Y	AREA Z	INERTIA X	INERTIA Y	INERTIA Z	FIBER DISTANCE DY	FIBER DISTANCE DZ
1	.4000+01	.5000+02	.1000+01	.1000+01	-.1000+01	.1000+01	.1000+01	.1000+01

BEAM ELEMENT PARAMETERS

BEAM NO	NODES I	J	K	MATL NO	GEOM NO	ELEM LOADS	SPEC CODE	END CODES I	J
1	1	2	8	1	1	0	0	0	0
2	2	3	8	1	1	0	0	0	0
3	3	4	8	1	1	0	0	0	0
4	4	5	8	1	1	0	0	0	0
5	5	6	8	1	1	0	0	0	0
6	6	7	8	1	1	0	0	0	0

```
TOTAL NUMBER OF EQUATIONS        =   34
BANDWIDTH                        =   12
NUMBER OF EQUATIONS IN A BLOCK   =   34
NUMBER OF BLOCKS                 =    1
```

.....NODAL POINT LOADS

NODE NO.	APPLIED LOADS					
	RX	RY	RZ	MX	MY	MZ
4	.000	1.000	1.000	.000	.000	.000

BEAM STRESSES

BEAM NO.	NODE I / NODE J	AXIAL STRESS	2 BENDING STRESSES (+3 DIR)	2 BENDING STRESSES (-3 DIR)	3 BENDING STRESSES (+2 DIR)	3 BENDING STRESSES (-2 DIR)	AVERAGE SHEAR STRESSES (+2 DIR)	AVERAGE SHEAR STRESSES (+3 DIR)
	FOR PRESSURE STABILIZED BEAM ELEMENTS STRESSES ARE MEMBRANE TYPE WITH UNITS FORCE/UNIT LENGTH							
1	1	.0000	-.0000	.0000	-.0000	.0000	-.0199	-.0199
	2	.0000	.0215	-.0215	.0215	-.0215	.0199	.0199
2	2	.0000	.0215	-.0215	.0215	-.0215	-.0199	-.0199
	3	.0000	.0561	-.0561	.0561	-.0561	.0199	.0199
3	3	.0000	.0561	-.0561	.0561	-.0561	-.0199	-.0199
	4	.0000	.1250	-.1250	.1250	-.1250	.0199	.0199
4	4	.0000	.1250	-.1250	.1250	-.1250	.0199	.0199
	5	.0000	.0561	-.0561	.0561	-.0561	-.0199	-.0199
5	5	.0000	.0561	-.0561	.0561	-.0561	.0199	.0199
	6	.0000	.0215	-.0215	.0215	-.0215	-.0199	-.0199
6	6	.0000	.0215	-.0215	.0215	-.0215	.0199	.0199
	7	.0000	-.0000	.0000	-.0000	.0000	-.0199	-.0199

BEAM FORCES AND MOMENTS

BEAM NO.	(NODE I) (NODE J) AXIAL R1	SHEAR R2	SHEAR R3	TORSION M1	BENDING M2	BENDING M3
1	.000	-.500+00	-.500+00	.000	.596-07	-.596-07
	.000	.500+00	.500+00	.000	.108+01	-.108+01
2	.000	-.500+00	-.500+00	.000	-.108+01	.108+01
	.000	.500+00	.500+00	.000	.232+01	-.282+01
3	.000	-.500+00	-.500+00	.000	-.282+01	.282+01
	.000	.500+00	.500+00	.000	.628+01	-.628+01
4	.000	.500+00	.500+00	.000	-.628+01	.628+01
	.000	-.500+00	-.500+00	.000	.282+01	-.282+01
5	.000	.500+00	.500+00	.000	-.282+01	.282+01
	.000	-.500+00	-.500+00	.000	.108+01	-.108+01
6	.000	.500+00	.500+00	.000	-.108+01	.108+01
	.000	-.500+00	-.500+00	.000	-.238-06	.238-06

TOTAL NODAL
DISPLACEMENTS AND ROTATIONS

NODE	X	Y	Z	XX	YY	ZZ
1	.000	.000	.000	.00	-.32-03	.32-03
2	.000	.710-02	.710-02	.00	-.29-03	.29-03
3	.000	.137-01	.137-01	.00	-.21-03	.21-03
4	.000	.189-01	.189-01	.00	-.27-11	.27-11
5	.000	.137-01	.137-01	.00	.21-03	-.21-03
6	.000	.710-02	.710-02	.00	.29-03	-.29-03
7	.000	.000	.000	.00	.32-03	-.32-03
8	.000	.000	.000	.00	.00	.00

N O D A L   P O S I T I O N S

NODE	X	Y	Z
1	.000	.000	.000
2	20.000	.007	.007
3	40.000	.014	.014
4	60.000	.019	.019
5	80.000	.014	.014
6	100.000	.007	.007
7	120.000	.000	.000
8	.000	10.000	.000

APPENDIX C

INPUT DATA AND RESULTS FOR ARCH CHECK CASE

INPUT DATA

```
      ARCH CHECK CASE FOR P. B. FINITE ELEMENT
        20    1    0    0    0          .0000     .0000     .000001101000
         1    1    1    1    0    0    0      .0000 -120.0000     .0000    0    0
         2    1    0    0    0    0    0      .0000 -118.1772   20.8380    0    0
         3    1    0    0    0    0    0      .0000 -112.7628   41.0424    0    0
         4    1    0    0    0    0    0      .0000 -103.9224   60.0000    0    0
         5    1    0    0    0    0    0      .0000  -91.9248   77.1336    0    0
         6    1    0    0    0    0    0      .0000  -77.1336   91.9248    0    0
         7    1    0    0    0    0    0      .0000  -60.0000  103.9224    0    0
         8    1    0    0    0    0    0      .0000  -41.0424  112.7628    0    0
         9    1    0    0    0    0    0      .0000  -20.8380  118.1772    0    0
        10    1    0    0    0    0    0      .0000     .0000  120.0000    0    0
        11    1    0    0    0    0    0      .0000   20.8380  118.1772    0    0
        12    1    0    0    0    0    0      .0000   41.0424  112.7628    0    0
        13    1    0    0    0    0    0      .0000   60.0000  103.9224    0    0
        14    1    0    0    0    0    0      .0000   77.1336   91.9248    0    0
        15    1    0    0    0    0    0      .0000   91.9248   77.1336    0    0
        16    1    0    0    0    0    0      .0000  103.9224   60.0000    0    0
        17    1    0    0    0    0    0      .0000  112.7628   41.0424    0    0
        18    1    0    0    0    0    0      .0000  118.1772   20.8380    0    0
        19    1    1    1    0    0    0      .0000  120.0000     .0000    0    0
        20    1    1    1    1    1    1      .0000     .0000     .0000    0    0
         1   18    1    0    1
         1 2100.0000    9.9375
         1    4.0000   50.0000    1.000     1.0000     -1.00      1.00001.0001.000
         1    1    2   20    1    1    0    0    0    0    0    1
         2    2    3   20    1    1    0    0    0    0    0    1
         3    3    4   20    1    1    0    0    0    0    0    1
         4    4    5   20    1    1    0    0    0    0    0    1
         5    5    6   20    1    1    0    0    0    0    0    1
         6    6    7   20    1    1    0    0    0    0    0    1
         7    7    8   20    1    1    0    0    0    0    0    1
         8    8    9   20    1    1    0    0    0    0    0    1
         9    9   10   20    1    1    0    0    0    0    0    1
        10   10   11   20    1    1    0    0    0    0    0    1
        11   11   12   20    1    1    0    0    0    0    0    1
        12   12   13   20    1    1    0    0    0    0    0    1
        13   13   14   20    1    1    0    0    0    0    0    1
        14   14   15   20    1    1    0    0    0    0    0    1
        15   15   16   20    1    1    0    0    0    0    0    1
        16   16   17   20    1    1    0    0    0    0    0    1
        17   17   18   20    1    1    0    0    0    0    0    1
        18   18   19   20    1    1    0    0    0    0    0    1
        10          .0000     .0000   -1.0000     .0000     .0000     .0000
```

RESULTS

```
**************************************************
*                                                *
*        P R O G R A M   N O N F E S A           *
*                                                *
**************************************************
*                                                *
* NONLINEAR FINITE ELEMENT STRUCTURAL ANALYSIS   *
* PROGRAM FOR DETERMINING DEFLECTIONS AND        *
* STRESSES OF TENT STRUCTURES.  THE PROGRAM IS A *
* GENERAL PROGRAM CAPABLE OF ASSEMBLING MANY     *
* DIFFERENT ELEMENT TYPES.  THE PRESENT VERSION  *
* (MARCH,1975) CONTAINS ELEMENTS TYPES-          *
*                1. LINEAR BEAM ELEMENT          *
*                   1.1 LINEAR BEAM              *
*                   1.2 MODIFIED BEAM            *
*                2. GEOMETRIC NONLINEAR MEMBRANE *
*                   2.1 LINEAR MATERIAL          *
*                   2.2 NONLINEAR MATERIAL       *
*                   2.3 MATERIAL REFERENCE PLANE *
*                3. LINEAR TRUSS ELEMENT         *
*                   3.1 TRUSS(PRE-LOAD)          *
*                   3.2 GUY LINE(TENSILE)        *
*                   3.3 COMPRESSIVE BAR          *
*                                                *
**************************************************
*                                                *
* PROGRAM DEVELOPED AT BOLT BERANEK AND NEWMAN   *
* CAMBRIDGE, MASS. BY J.C. OCALLAHAN.            *
*                                                *
**************************************************
```

```
ARCH CHECK CASE FOR P. B. FINITE ELEMENT

NUMBER OF NODAL POINTS    =   20

NUMBER OF ELEMENT TYPES   =    1

MAXIMUM NUMBER OF CYCLES =    0

NUMBER OF LOAD STEPS      =    0

CONVERGENCE TOLERANCE     =   .000

STARTING MEMBRANE TENSION=   .000

DERIVATIVE SPACING        =   .100+00

PRINT CONTROL
   (0--OFF, 1--ON, EXCEPT FOR DISPLACEMENTS)
   1. DELTA DISPLACEMENTS  = 0
   2. ELEMENT STRESSES     = 1
   3. BEAM FORCES          = 1
   4. DISPLACEMENTS(1--OFF)= 0
   5. NODAL POSITIONS      = 1
   6. DEBUG-MEMBRANE       = 0
        ELEMENT STIFFNESS
   7. DEBUG-MEMBRANE       = 0
        GEOMETRIC PROP
   8. DEBUG-MEMBRANE       = 0
        MATERIAL COEFS
```

NODAL POINT INPUT DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	NODAL POINT COORDINATES X	Y	Z		
1	1	1	1	0	0	0	.000	-120.000	.000	0	0
2	1	0	0	0	0	0	.000	-118.177	20.838	0	0
3	1	0	0	0	0	0	.000	-112.763	41.042	0	0
4	1	0	0	0	0	0	.000	-103.922	60.000	0	0
5	1	0	0	0	0	0	.000	-91.925	77.134	0	0
6	1	0	0	0	0	0	.000	-77.134	91.925	0	0
7	1	0	0	0	0	0	.000	-60.000	103.922	0	0
8	1	0	0	0	0	0	.000	-41.042	112.763	0	0
9	1	0	0	0	0	0	.000	-20.838	118.177	0	0
10	1	0	0	0	0	0	.000	.000	120.000	0	0
11	1	0	0	0	0	0	.000	20.838	118.177	0	0
12	1	0	0	0	0	0	.000	41.042	112.763	0	0
13	1	0	0	0	0	0	.000	60.000	103.922	0	0
14	1	0	0	0	0	0	.000	77.134	91.925	0	0
15	1	0	0	0	0	0	.000	91.925	77.134	0	0
16	1	0	0	0	0	0	.000	103.922	60.000	0	0
17	1	0	0	0	0	0	.000	112.763	41.042	0	0
18	1	0	0	0	0	0	.000	118.177	20.838	0	0
19	1	1	1	0	0	0	.000	120.000	.000	0	0
20	1	1	1	1	1	1	.000	.000	.000	0	0

GENERATED NODAL DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	NODAL POINT COORDINATES X	Y	Z
1	1	1	1	0	0	0	.000	-120.000	.000
2	1	0	0	0	0	0	.000	-118.177	20.838
3	1	0	0	0	0	0	.000	-112.763	41.042
4	1	0	0	0	0	0	.000	-103.922	60.000
5	1	0	0	0	0	0	.000	-91.925	77.134
6	1	0	0	0	0	0	.000	-77.134	91.925
7	1	0	0	0	0	0	.000	-60.000	103.922
8	1	0	0	0	0	0	.000	-41.042	112.763
9	1	0	0	0	0	0	.000	-20.838	118.177
10	1	0	0	0	0	0	.000	.000	120.000
11	1	0	0	0	0	0	.000	20.838	118.177
12	1	0	0	0	0	0	.000	41.042	112.763
13	1	0	0	0	0	0	.000	60.000	103.922
14	1	0	0	0	0	0	.000	77.134	91.925
15	1	0	0	0	0	0	.000	91.925	77.134
16	1	0	0	0	0	0	.000	103.922	60.000
17	1	0	0	0	0	0	.000	112.763	41.042
18	1	0	0	0	0	0	.000	118.177	20.838
19	1	1	1	0	0	0	.000	120.000	.000
20	1	1	1	1	1	1	.000	.000	.000

EQUATION NUMBERS

N	X	Y	Z	XX	YY	ZZ
1	0	0	0	1	2	3
2	0	4	5	6	7	8
3	0	9	10	11	12	13

4	0	14	15	16	17	18
5	0	19	20	21	22	23
6	0	24	25	26	27	28
7	0	29	30	31	32	33
8	0	34	35	36	37	38
9	0	39	40	41	42	43
10	0	44	45	46	47	48
11	0	49	50	51	52	53
12	0	54	55	56	57	58
13	0	59	60	61	62	63
14	0	64	65	66	67	68
15	0	69	70	71	72	73
16	0	74	75	76	77	78
17	0	79	80	81	82	83
18	0	84	85	86	87	88
19	0	0	0	89	90	91
20	0	0	0	0	0	0

```
T H R E E   D I M E N S I O N A L   B E A M   E L E M E N T S

     NUMBER OF BEAMS                     =    18
     NUMBER OF GEOMETRIC PROPERTY SETS=     1
     NUMBER OF FIXED END FORCE SETS   =     0
     NUMBER OF MATERIALS                 =     1
```

MATERIAL	YOUNG S MODULUS	POISSON S RATIO
1	.21000+04	9.93750

BEAM GEOMETRIC PROPERTIES

ELEMENT TYPE	AREA X	AREA Y	AREA Z	INERTIA X	INERTIA Y	INERTIA Z	FIBER DISTANCE DY	FIBER DISTANCE DZ
1	.4000+01	.5000+02	.1000+01	.1000+01	-.1000+01	.1000+01	.1000+01	.1000+01

BEAM ELEMENT PARAMETERS

BEAM NO	NODES I	J	K	MATL NO	GEOM NO	ELEM LOADS	SPEC CODE	END CODES I	J
1	1	2	20	1	1	0	0	0	0
2	2	3	20	1	1	0	0	0	0
3	3	4	20	1	1	0	0	0	0
4	4	5	20	1	1	0	0	0	0
5	5	6	20	1	1	0	0	0	0
6	6	7	20	1	1	0	0	0	0
7	7	8	20	1	1	0	0	0	0
8	8	9	20	1	1	0	0	0	0
9	9	10	20	1	1	0	0	0	0
10	10	11	20	1	1	0	0	0	0
11	11	12	20	1	1	0	0	0	0
12	12	13	20	1	1	0	0	0	0
13	13	14	20	1	1	0	0	0	0
14	14	15	20	1	1	0	0	0	0
15	15	16	20	1	1	0	0	0	0
16	16	17	20	1	1	0	0	0	0
17	17	18	20	1	1	0	0	0	0
18	18	19	20	1	1	0	0	0	0

TOTAL NUMBER OF EQUATIONS = 91
BANDWIDTH = 10
NUMBER OF EQUATIONS IN A BLOCK = 91
NUMBER OF BLOCKS = 1

.....NODAL POINT LOADS

NODE NO.			APPLIED LOADS			
	RX	RY	RZ	MX	MY	MZ
10	.000	.000	-1.000	.000	.000	.000

..........................B E A M   S T R E S S E S..................................

FOR PRESSURE STABILIZED BEAM ELEMENTS STRESSES ARE
MEMBRANE TYPE WITH UNITS FORCE/UNIT LENGTH

BEAM NO.	NODE I / NODE J	AXIAL STRESS	2 BENDING STRESSES (+3 DIR)	2 BENDING STRESSES (-3 DIR)	3 BENDING STRESSES (+2 DIR)	3 BENDING STRESSES (-2 DIR)	AVERAGE SHEAR STRESSES (+2 DIR)	AVERAGE SHEAR STRESSES (+3 DIR)
1	1	-.0209	.0000	.0000	.0000	-.0000	.0103	.0000
	2	-.0209	.0000	.0000	-.0178	.0178	-.0103	.0000
2	2	-.0224	.0000	.0000	-.0178	.0178	.0066	.0000
	3	-.0224	.0000	.0000	-.0261	.0261	-.0066	.0000
3	3	-.0232	.0000	.0000	-.0261	.0261	.0026	.0000
	4	-.0232	.0000	.0000	-.0293	.0293	-.0026	.0000
4	4	-.0233	.0000	.0000	-.0293	.0293	-.0015	.0000
	5	-.0233	.0000	.0000	-.0291	.0291	.0015	.0000
5	5	-.0226	.0000	.0000	-.0291	.0291	-.0055	.0000
	6	-.0226	.0000	.0000	-.0256	.0256	.0055	.0000
6	6	-.0213	.0000	.0000	-.0256	.0256	-.0093	.0000
	7	-.0213	.0000	.0000	-.0175	.0175	.0093	.0000
7	7	-.0194	.0000	.0000	-.0175	.0175	-.0129	.0000
	8	-.0194	.0000	.0000	-.0009	.0009	.0129	.0000
8	8	-.0169	.0000	.0000	-.0009	.0009	-.0161	.0000
	9	-.0169	.0000	.0000	.0331	-.0331	.0161	.0000
9	9	-.0138	.0000	.0000	.0331	-.0331	-.0188	.0000
	10	-.0138	.0000	.0000	.1046	-.1046	.0188	.0000
10	10	-.0138	.0000	.0000	.1046	-.1046	.0188	.0000
	11	-.0138	.0000	.0000	.0331	-.0331	-.0188	.0000
11	11	-.0169	.0000	.0000	.0331	-.0331	.0161	.0000
	12	-.0169	.0000	.0000	-.0009	.0009	-.0161	.0000
12	12	-.0194	.0000	.0000	-.0009	.0009	.0129	.0000
	13	-.0194	.0000	.0000	-.0175	.0175	-.0129	.0000
13	13	-.0213	.0000	.0000	-.0175	.0175	.0093	.0000
	14	-.0213	.0000	.0000	-.0256	.0256	-.0093	.0000
14	14	-.0226	.0000	.0000	-.0256	.0256	.0055	.0000
	15	-.0226	.0000	.0000	-.0291	.0291	-.0055	.0000
15	15	-.0233	.0000	.0000	-.0291	.0291	.0015	.0000
	16	-.0233	.0000	.0000	-.0293	.0293	-.0015	.0000
16	16	-.0232	.0000	.0000	-.0293	.0293	-.0026	.0000
		-.0232	.0000	.0000	-.0261	.0261	.0026	.0000

17	17	-.0224	.0000	.0000	-.0261	.0261	-.0066	.0000
	18	-.0224	.0000	.0000	-.0178	.0178	.0066	.0000
18	18	-.0209	.0000	.0000	-.0178	.0178	-.0103	.0000
	19	-.0209	.0000	.0000	.0000	-.0000	.0103	.0000

BEAM FORCES AND MOMENTS

BEAM NO.	(NODE I) (NODE J) AXIAL R1	SHEAR R2	SHEAR R3	TORSION M1	BENDING M2	BENDING M3
1	.525+00	.260+00	.000	.000	.000	.596-07
	-.525+00	-.260+00	.000	.000	.000	.895+00
2	.562+00	.165+00	.000	.000	.000	-.895+00
	-.562+00	-.165+00	.000	.000	.000	.131+01
3	.582+00	.648-01	.000	.000	.000	-.131+01
	-.582+00	-.648-01	.000	.000	.000	.147+01
4	.584+00	-.372-01	.000	.000	.000	-.147+01
	-.584+00	.372-01	.000	.000	.000	.146+01
5	.569+00	-.138+00	.000	.000	.000	-.146+01
	-.569+00	.138+00	.000	.000	.000	.129+01
6	.535+00	-.235+00	.000	.000	.000	-.129+01
	-.536+00	.235+00	.000	.000	.000	.880+00
7	.487+00	-.324+00	.000	.000	.000	-.880+00
	-.487+00	.324+00	.000	.000	.000	.465-01
8	.424+00	-.404+00	.000	.000	.000	-.465-01
	-.424+00	.404+00	.000	.000	.000	-.166+01
9	.347+00	-.472+00	.000	.000	.000	.166+01
	-.347+00	.472+00	.000	.000	.000	-.526+01
10	.347+00	.472+00	.000	.000	.000	.526+01
	-.347+00	-.472+00	.000	.000	.000	-.166+01
11	.424+00	.404+00	.000	.000	.000	.166+01
	-.424+00	-.404+00	.000	.000	.000	.465-01
12	.487+00	.324+00	.000	.000	.000	-.465-01
	-.487+00	-.324+00	.000	.000	.000	.880+00
13	.536+00	.235+00	.000	.000	.000	-.880+00
	-.536+00	-.235+00	.000	.000	.000	.129+01

14	.569+00	.138+00	.000	.000	.000	-.129+01
	-.569+00	-.138+00	.000	.000	.000	.146+01
15	.584+00	.372-01	.000	.000	.000	-.146+01
	-.584+00	-.372-01	.000	.000	.000	.147+01
16	.582+00	-.648-01	.000	.000	.000	-.147+01
	-.582+00	.648-01	.000	.000	.000	.131+01
17	.562+00	-.165+00	.000	.000	.000	-.131+01
	-.562+00	.165+00	.000	.000	.000	.895+00
18	.525+00	-.260+00	.000	.000	.000	-.895+00
	-.525+00	.260+00	.000	.000	.000	-.209-06

TOTAL NODAL DISPLACEMENTS AND ROTATIONS

NODE	X	Y	Z	XX	YY	ZZ
1	.000	.000	.000	.14-03	.00	.00
2	.000	-.362-02	.108-03	.12-03	.00	.00
3	.000	-.601-02	.518-03	.71-04	.00	.00
4	.000	-.697-02	.711-03	.57-05	.00	.00
5	.000	-.660-02	.170-03	-.63-04	.00	.00
6	.000	-.523-02	-.152-02	-.13-03	.00	.00
7	.000	-.335-02	-.457-02	-.18-03	.00	.00
8	.000	-.152-02	-.895-02	-.20-03	.00	.00
9	.000	-.298-03	-.142-01	-.16-03	.00	.00
10	.000	.175-07	-.192-01	.11-09	.00	.00
11	.000	.298-03	-.142-01	.16-03	.00	.00
12	.000	.152-02	-.895-02	.20-03	.00	.00
13	.000	.335-02	-.457-02	.18-03	.00	.00
14	.000	.523-02	-.152-02	.13-03	.00	.00
15	.000	.660-02	.170-03	.63-04	.00	.00
16	.000	.697-02	.711-03	-.57-05	.00	.00
17	.000	.601-02	.518-03	-.71-04	.00	.00
18	.000	.362-02	.108-03	-.12-03	.00	.00
19	.000	.000	.000	-.14-03	.00	.00
20	.000	.000	.000	.00	.00	.00

NODAL POSITIONS

NODE	X	Y	Z
1	.000	-120.000	.000
2	.000	-118.181	20.838
3	.000	-112.769	41.043
4	.000	-103.929	60.001
5	.000	-91.931	77.134
6	.000	-77.139	91.923
7	.000	-60.003	103.918
8	.000	-41.044	112.754
9	.000	-20.838	118.163
10	.000	.000	119.981
11	.000	20.838	118.163
12	.000	41.044	112.754
13	.000	60.003	103.918
14	.000	77.139	91.923
15	.000	91.931	77.134
16	.000	103.929	60.001
17	.000	112.769	41.043
18	.000	118.181	20.838
19	.000	120.000	.000
20	.000	.000	.000

SYMBOLS

a	Beam cross-section [illegible]s
C	shear stiffness
C_{11}	elastic modulus
C_{33}	shear modulus
D	bending stiffness
E	axial stiffness
G	torsional stiffness
h_i	shape functions used in displacement approximations
K_{ij}	submatrices of the 8 x 8 bending stiffness matrix
L	beam length
P	pressure parameter
p	pressure
Q	vector of generalized forces
S	4 x 4 bending stiffness matrix
S_{ij}	elements of the matrix S
u	axial displacement
U_1, U_2	nodal axial displacements
v, w	bending displacements
V_1, V_2, W_1, W_2	nodal bending displacements
x	axial coordinate
Δ_1, Δ_2	vector defined by equation (9)

θ, ϕ, ψ	cross-section rotations
θ_1, θ_2	nodal torsional rotations
$\phi_1, \phi_2, \psi_1, \psi_2$	nodal bending rotations
$(\ )'$	denotes differentiation with respect to x